Repetition §3.1 Gauss-Algorithmus

LGS:
$$\boxed{\begin{array}{rcl} a+b+c+d &=& 2 \\ 2a \quad\; +c+d &=& 3 \\ b+c \quad\; &=& 1 \end{array}}$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

-2

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -1 & -1 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & -1 & -1 \end{array}\right)$$

2

$$\left(\begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right)$$

-1 Zeilenstufenform

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right)$$

-1

$$\left(\begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{array}\right)$$

$$\left\{\begin{array}{l} a + d = 1 \\ b + d = 0 \\ c - d = 1 \end{array}\right\}$$

$$\iff \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1-d \\ -d \\ 1+d \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + d \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

partikuläre Lösung

Basis des Lösungsraums des homogenen LGS.

$$
\left(\begin{array}{cccc|c}
1 & 1 & 1 & 1 & 2 \\
2 & 0 & 1 & 1 & 3 \\
0 & 1 & 1 & 0 & 1
\end{array}\right)
\quad -2,\; -1
$$

$$
\left(\begin{array}{cccc|c}
0 & -2 & -1 & -1 & -1
\end{array}\right)
\quad +2,\; -1
$$

$$
\left(\begin{array}{cccc|c}
0 & 0 & 1 & -1 & 1
\end{array}\right)
$$

$$
\boxed{\left(\begin{array}{cccc|c}
1 & 0 & 0 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & -1 & 1
\end{array}\right)}
$$

$$
\left(\begin{array}{cccc|c}
0 & 0 & 0 & -1 & 0
\end{array}\right)
$$

$$
\Rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}
$$

Repetition § 3.8 LR-Zerlegung

$$A = \begin{pmatrix} 0 & 0 & 3 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & 2 & -3 & 3 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

Gesucht:
- P Permutationsmatrix
- L linke Dreiecksmatrix
- R rechte Dreiecksmatrix

mit $\boxed{PA = LR}$

$K = \mathbb{Q}$ oder $\mathbb{R}$.

$$P_1 := \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \Rightarrow P_1 A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 2 & -3 & 3 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

$$L_1 := \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ 1 & & 1 & \\ 1 & & & 1 \end{pmatrix} \Rightarrow L_1^{-1} \cdot P_1 \cdot A = \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ -1 & & 1 & \\ -1 & & & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 2 & -3 & 3 \\ 1 & 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$$P_2 := \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \Rightarrow P_2 L_1^{-1} P_1 A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$$L_2 := \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & 1 & & 1 \end{pmatrix} \Rightarrow \underbrace{L_2^{-1} P_2 L_1^{-1} P_1 A}_{} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & -1 & & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & -2 \end{pmatrix}$$

$$L_3 := \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 1 & 1 \end{pmatrix} \Rightarrow L_3^{-1} \cdot (\ldots) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & -1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & -2 \end{pmatrix} = \boxed{\underbrace{\begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}}_{= R}}$$

$$\Rightarrow L_3^{-1} \cdot L_2^{-1} P_2 L_1^{-1} P_1 \cdot A = R$$

$$\| $$

$$\underbrace{L_3^{-1} L_2^{-1} P_2 L_1^{-1} P_2^{-1}} \cdot \underbrace{P_2 P_1} \cdot A$$

$$\Rightarrow \underbrace{P_2 P_1}_{= P} \cdot A = \underbrace{P_2 L_1 P_2^{-1} L_2 L_3}_{= L} \cdot R$$

$$\boxed{P =} \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}$$

$$\boxed{L =} \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ 0 & 1 & & \\ 1 & & 1 & \\ 1 & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & 1 & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & & & \\ 1 & 1 & & \\ 0 & & 1 & \\ 1 & & & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} = \boxed{\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}}$$