

# Repetition §3.1 Gauss-Algorithmus

LGS:

$$\begin{cases} a+b+c+d=2 \\ 2a+c+d=3 \\ b+c=1 \end{cases}$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 2 & 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} \uparrow -2 \\ \\ \end{array}$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & -2 & -1 & -1 & -1 \\ 0 & 1 & 1 & 0 & 1 \end{array} \right) \begin{array}{l} \\ \uparrow \\ \end{array}$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & -2 & -1 & -1 & -1 \end{array} \right) \begin{array}{l} \\ \downarrow 2 \\ \end{array}$$

$$\left( \begin{array}{cccc|c} 1 & 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{array} \right) \begin{array}{l} \uparrow -1 \\ \\ \end{array} \text{ Zeilenstufenform}$$

$$\left( \begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 1 \end{array} \right) \begin{array}{l} \\ \uparrow -1 \\ \end{array}$$

$$\left( \begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 1 \end{array} \right)$$

$$\begin{cases} a+d=1 \\ b+d=0 \\ c-d=1 \end{cases}$$

$$\Leftrightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1-d \\ -d \\ 1+d \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + d \cdot \begin{pmatrix} -1 \\ -1 \\ 1 \\ 1 \end{pmatrix}$$

partikuläre Lösung  
Basis des Lösungsraums  
des homogenen LGS.

$$\begin{array}{cccc|c}
 \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{2} \\
 \cancel{2} & \cancel{0} & \cancel{1} & \cancel{1} & \cancel{3} \\
 \cancel{0} & \cancel{1} & \cancel{1} & \cancel{0} & \cancel{1} \\
 \hline
 0 & -2 & -1 & -1 & -1 \\
 \hline
 0 & 0 & 1 & -1 & 1
 \end{array}$$

Annotations:
 

- Green arrows:  $\downarrow -2$  (from row 1 to row 2),  $\downarrow +2$  (from row 3 to row 4).
- Green brackets:  $\left. \begin{array}{l} \text{Row 1} \\ \text{Row 2} \end{array} \right\} -1$  and  $\left. \begin{array}{l} \text{Row 3} \\ \text{Row 4} \end{array} \right\} -1$ .

$$\begin{array}{cccc|c}
 1 & 0 & 0 & 1 & 1 \\
 \hline
 0 & 1 & 0 & 1 & 0 \\
 0 & 0 & 1 & -1 & 1 \\
 \hline
 0 & 0 & 0 & -1 & 0
 \end{array}$$

Annotations:
 

- Red box: Encloses the first three rows.
- Green box: Encloses the first three columns of the first three rows.
- Orange ovals: Enclose the fourth and fifth columns of the first three rows.

$$\Rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} + \lambda \cdot \begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

# Repetition § 3.8 LR-Zerlegung

$$A = \begin{pmatrix} 0 & 0 & 3 & 0 \\ 1 & 1 & -1 & 0 \\ 1 & 2 & -3 & 3 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

Gesamt: P Permutationsmatrix  
L linke Dreiecksmatrix  
R rechte Dreiecksmatrix

mit  $PA = LR$

$K = \mathbb{Q}$  oder  $\mathbb{R}$ .

$$P_1 = \left( \begin{array}{c|c} 1 & \\ \hline & 1 \\ & & 1 \end{array} \right)$$

$$\Rightarrow P_1 A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 2 & -3 & 3 \\ 1 & 2 & 0 & 1 \end{pmatrix}$$

$$L_1 = \left( \begin{array}{c|c} 1 & \\ \hline 0 & 1 \\ 1 & & 1 \\ 1 & & & 1 \end{array} \right)$$

$$\Rightarrow L_1^{-1} \cdot P_1 A = \left( \begin{array}{c|c} 1 & \\ \hline 0 & 1 \\ -1 & & 1 \\ -1 & & & 1 \end{array} \right) \cdot \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 1 & 2 & -3 & 3 \\ 1 & 2 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$$P_2 = \left( \begin{array}{c|c|c} 1 & & \\ \hline & 1 & \\ & & 1 \\ & & & 1 \end{array} \right)$$

$$\Rightarrow P_2 L_1^{-1} P_1 A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

$$L_2 := \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \Rightarrow L_2^{-1} P_2^{-1} L_1^{-1} P_1 A = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & -2 \end{pmatrix}$$

$$L_3 := \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \Rightarrow L_3^{-1} \cdot (\dots) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 3 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -2 \end{pmatrix}$$

$R$

$$\Rightarrow L_3^{-1} \cdot L_2^{-1} P_2^{-1} L_1^{-1} P_1 A = R$$

$$\underbrace{L_3^{-1} L_2^{-1} P_2^{-1} L_1^{-1} P_1^{-1}}_{P_2^{-1} P_1^{-1}} \cdot \underbrace{P_2 P_1 A}$$

$$\Rightarrow \underbrace{P_2 P_1}_{P^{-1}} \cdot A = \underbrace{P_2^{-1} L_1^{-1} P_2^{-1} L_2^{-1} L_3^{-1}}_{L^{-1}} \cdot R$$

$$P = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$L = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$